

Problem 2.34

[Difficulty: 3]

2.34 A flow is described by velocity field $\vec{V} = a\hat{i} + bx\hat{j}$, where $a = 2 \text{ m/s}$ and $b = 1 \text{ s}^{-1}$. Coordinates are measured in meters. Obtain the equation for the streamline passing through point (2, 5). At $t = 2 \text{ s}$, what are the coordinates of the particle that passed through point (0, 4) at $t = 0$? At $t = 3 \text{ s}$, what are the coordinates of the particle that passed through point (1, 4.25) 2 s earlier? What conclusions can you draw about the pathline, streamline, and streakline for this flow?

Given: Velocity field

Find: Equation for streamline through point (2.5); coordinates of particle at $t = 2 \text{ s}$ that was at (0,4) at $t = 0$; coordinates of particle at $t = 3 \text{ s}$ that was at (1,4.25) at $t = 1 \text{ s}$; compare pathline, streamline, streakline

Solution:

Governing equations: For streamlines $\frac{v}{u} = \frac{dy}{dx}$ For pathlines $u_p = \frac{dx}{dt}$ $v_p = \frac{dy}{dt}$

Assumption: 2D flow

Given data $a = 2 \frac{\text{m}}{\text{s}}$ $b = 1 \frac{1}{\text{s}}$ $x_0 = 2$ $y_0 = 5$ $x = 1$ $x = x$

For streamlines $\frac{v}{u} = \frac{dy}{dx} = \frac{b \cdot x}{a}$

So, separating variables $\frac{a}{b} \cdot dy = x \cdot dx$

Integrating $\frac{a}{b} \cdot (y - y_0) = \frac{1}{2} \cdot (x^2 - x_0^2)$

The solution is then $y = y_0 + \frac{b}{2 \cdot a} \cdot (x^2 - x_0^2) = \frac{x^2}{4} + 4$

Hence for pathlines $u_p = \frac{dx}{dt} = a$ $v_p = \frac{dy}{dt} = b \cdot x$

Hence $dx = a \cdot dt$ $dy = b \cdot x \cdot dt$

Integrating $x - x_0 = a \cdot (t - t_0)$ $dy = b \cdot [x_0 + a \cdot (t - t_0)] \cdot dt$

$$y - y_0 = b \cdot \left[x_0 \cdot (t - t_0) + \frac{a}{2} \cdot (t^2 - t_0^2) \right] - a \cdot t_0 \cdot (t - t_0)$$

The pathlines are $x = x_0 + a \cdot (t - t_0)$ $y = y_0 + b \cdot \left[x_0 \cdot (t - t_0) + \frac{a}{2} \cdot (t^2 - t_0^2) \right] - a \cdot t_0 \cdot (t - t_0)$

For a particle that was at $x_0 = 0$ m, $y_0 = 4$ m at $t_0 = 0$ s, at time $t = 2$ s we find the position is

$$x = x_0 + a \cdot (t - t_0) = 4 \text{ m} \quad y = y_0 + b \cdot \left[x_0 \cdot (t - t_0) + \frac{a}{2} \cdot \left((t^2 - t_0^2) \right) - a \cdot t_0 \cdot (t - t_0) \right] = 10 \text{ m}$$

For a particle that was at $x_0 = 1$ m, $y_0 = 4.25$ m at $t_0 = 1$ s, at time $t = 3$ s we find the position is

$$x = x_0 + a \cdot (t - t_0) = 5 \text{ m} \quad y = y_0 + b \cdot \left[x_0 \cdot (t - t_0) + \frac{a}{2} \cdot \left((t^2 - t_0^2) \right) - a \cdot t_0 \cdot (t - t_0) \right] = 10.5 \text{ m}$$

For this steady flow streamlines, streaklines and pathlines coincide; the particles referred to are the same particle!

